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Non-universal critical dynamics of the one-dimensional Potts models

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Abstract. The one-dimensional kinetic Potts models are studied for both Glauber and Kawasaki dynamics with different transition rates. Rigorous lower bounds for the dynamical exponent (z) are obtained from the initial response rate and the scaling hypothesis for the relaxation time. It is shown that the z exponent is non-universal for several periodic models with a basic unit cell ($J_1, J_2, \ldots, J_M$) containing different couplings constants. The critical dynamics of the alternating bond models is also studied by a simple argument about the movement of domain walls.

1. Introduction

One of the simplest models exhibiting non-trivial dynamic behaviour is the kinetic Ising model (Glauber 1963). It consists of an Ising model with a stochastic dynamics. Near the critical temperature one speaks of critical dynamics. According to the dynamical scaling hypothesis (Hohenberg and Halperin 1977), when the critical temperature is approached the relaxation time of the system diverges as

$$\tau_k = \xi^z f(k\xi) \tag{1.1}$$

where ξ is the static correlation length, z is the critical dynamical exponent and k is the appropriate critical wavevector. One important point is the classification of the physical systems into dynamical universality classes. It turns out that the z exponent depends not only on static properties but also on dynamical aspects, such as the conservation laws entering the dynamics (Hohenberg and Halperin 1977).

In recent years the question of the universality of the z exponent in one dimension has been discussed in the literature. It is well known that the z exponent depends on the transition rates (Haake and Thol 1980), which can be chosen in several different forms once the detailed balance has been satisfied. More surprising is the non-universality due to the spatial non-uniformity of the interactions (Droz *et al* 1986a, Luscombe 1987, Angles d'Auriac and Rammal 1988). It is worth mentioning that this non-universal behaviour occurs for fixed transition rates, implying that the non-universality is related to intrinsic properties of the system (i.e. the interactions).

In this paper we consider the Q-state Potts kinetic model in one dimension (for a review of the static properties see Wu (1982)). It is, perhaps, the simplest generalisation

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of the kinetic Ising model. Some dynamical aspects of this model have already been discussed in the literature. In particular, it has been discussed that the z exponent depends on the transition rates and on the number of states (Q) (Lage 1985, Droz *et al* 1986b, Weir *et al* 1986). Here we investigate the dependence of the dynamic behaviour on the spatial non-uniformity of the interactions both for the relaxational dynamics (Glauber dynamics) and diffusive dynamics (Kawasaki dynamics). Unable to solve the equations of motion exactly, we obtain information about critical dynamics from the initial response rate of the order parameter (conventional theory) (Halperin 1973). Using the scaling hypothesis for the relaxation rate we are able to obtain a rigorous lower bound for the z exponent. Therefore we prove that z is non-universal for several models in which the basic cell contains different coupling constants $(J_1, J_2, J_3, \ldots, J_M)$. Naturally, the dynamical exponent depends also on the transition rates. For the alternating bond models $(J_1, J_2, J_1, J_2 \ldots)$, the z exponent is found also by the domain wall argument (Cordery *et al* 1981) for both dynamics.

This paper is organised as follows. In the next section we find the equilibrium properties of the Potts model in one dimension and discuss the dynamics (Glauber and Kawasaki dynamics). In § 3 we present in detail the initial response and derive the lower bounds. The z exponent for Glauber dynamics with several transition rates can be found in § 4. The non-universality of the dynamical exponent for Kawasaki dynamics is discussed in § 5. The domain wall argument is applied to the alternating bond models for both dynamics in § 6. The conclusions are presented in the last section.

2. The dynamic models

We consider the Q state Potts chain described by the following reduced Hamiltonian with N variables

$$\mathcal{H} = \sum_{i=1}^{N} K_i \delta_{q_i, q_{i+1}} \tag{2.1}$$

where $\delta_{q,q'}$ is the Kronecker delta, $K_i = J_i/(k_B T)$ and J_i is the next-nearest-neighbouring interaction couplings. The interaction couplings are periodically distributed with a basic cell $(J_1, J_2 \ldots J_M)$.

The equilibrium properties are easily found by the transfer matrix technique. The critical temperature is zero ($T_c = 0$) and the correlation length is given by

$$\xi^{-1} = -\lim_{(l-k)\to\infty} \left\{ \frac{1}{l-k} \sum_{i=1}^{l-k} \ln \frac{\exp(K_i) - 1}{\exp(K_i) + Q - 1} \right\} \qquad l > k. \tag{2.2}$$

Near the critical temperature, the correlation length depends only on the smallest interaction J_s of the basic cell, i.e.

$$\xi \sim Q^{-1} \exp(K_s) \qquad T \to 0. \tag{2.3}$$

If we evaluate the static critical exponents, we find that they are universal in the sense that they do not depend on the interactions.

The Glauber dynamics (Glauber 1963) is given by the master equation for $P(\{q\}, t)$, the density probability that the configuration $\{q_1, q_2 \ldots q_N\}$ is realised at time t:

$$\frac{\partial}{\partial t} P(\{q\}, t) = \sum_{i=1}^{N} \sum_{q_i'=1}^{Q} [w_i(\{q\}_i', \{q\}) P(\{q\}_i', t) - w_i(\{q\}, \{q\}_i') P(\{q\}, t)]. \tag{2.4}$$

Here $w_i(\{q\}'_i, \{q\})$ stands for the transition rate of the $\{q_1, q_2 \dots q'_i \dots q_N\}$ configuration to the $\{q_1, q_2 \dots q_i \dots q_N\}$ one. In order to assure the equilibrium distribution at long times, the transition rates must obey the detailed balance condition, namely

$$w_i(\{q\}'_i, \{q\}) P_{eq}(\{q\}'_i) = w_i(\{q\}, \{q\}'_i) P_{eq}(\{q\}) \tag{2.5}$$

where $P_{eq}(\{q\})$ is the equilibrium probability. This condition determines only partially the transition rates. So we can choose $w_i(\{q\}, \{q\}'_i)$ in several forms (Haake and Thol 1980). In the next sections we will consider two different rates.

The Kawasaki dynamics (Kawasaki 1972) has the order parameter (i.e. the magnetisation) conserved. It is described by the following master equation:

$$\frac{\partial}{\partial t} P(\{q\}, t) = \sum_{i=1}^{N} [W_i(\{q\}'_i, \{q\}) P(\{q\}'_i, t) - W_i(\{q\}, \{q\}'_i) P(\{q\}, t)] \tag{2.6}$$

where $W_i(\{q\}, \{q\}'_i)$ is the transition rate of the configuration $\{q_1, q_2 \dots q_i, q_{i+1} \dots q_N\}$ to the $\{q_1, q_2 \dots q_{i+1}, q_i \dots q_N\}$ one. Note that only the q_i and q_{i+1} states have exchanged. Again we impose the detailed balance condition. We have that

$$W_i(\{q\}'_i, \{q\}) P_{eq}(\{q\}'_i) = W_i(\{q\}, \{q\}'_i) P_{eq}(\{q\}). \tag{2.7}$$

Again the transition rates can be chosen in different ways. So we consider in § 5 the most used transition rates.

3. The initial response

The purpose of this section is to establish a rigorous lower bound for the relaxation time of the system as has been done for the Ising model (Halperin 1973). Firstly the Glauber dynamics will be considered.

Let us introduce a function ϕ, defined by

$$P(\{q\}, t) = P_{eq}(\{q\}) \phi(\{q\}, t). \tag{3.1}$$

Using this definition, the master equation (2.4) can be written as

$$\frac{\partial}{\partial t} \phi(\{q\}, t) = -D_q \phi(\{q\}, t) \tag{3.2}$$

where the D_q operator is defined by

$$D_q \phi(\{q\}, t) = \sum_{i=1}^{N} \sum_{q'_i=1}^{Q} w_i(\{q\}, \{q\}'_i)[\phi(\{q\}, t) - \phi(\{q\}'_i, t)]. \tag{3.3}$$

The formal solution of equation (3.2) is given by $\phi(\{q\}, t) = \exp[-D_q t]\phi(\{q\}, 0)$. From (3.3) we can see that D_q is a real operator and using the detailed balance is easy to show that it has the following properties:

$$\langle f^* D_q g \rangle_{eq} = \langle g D_q f^* \rangle_{eq} \qquad \langle g^* D_q g \rangle_{eq} \geq 0. \tag{3.4}$$

Here f and g are arbitrary functions of $\{q\}$. Thus the eigenvalues ν_i of the operator D_q are real and non-negative. We consider the time-dependent autocorrelation function of g

$$C_g(t) = \langle g^*[0] g[t] \rangle_{eq} - \langle g^*[0] g[\infty] \rangle_{eq} \tag{3.5}$$

where $g[t] = \exp(-D_q t) g$. It has a spectral representation of the form

$$C_g(t) = \int_{0^+}^{\infty} \varphi_g(\nu) \exp(-\nu t)\, d\nu \tag{3.6}$$

with $\varphi_g(\nu) \geqslant 0$ for all ν. We define the *characteristic time* (τ_g) and the *initial relaxation rate* (ν_g) for the variable g by

$$\tau_g = C_g(0)^{-1} \int_0^\infty C_g(t)\,\mathrm{d}t$$
$$\nu_g = -C_g(0)^{-1} \frac{\mathrm{d}}{\mathrm{d}t} C_g(t)\bigg|_{t=0}. \tag{3.7}$$

These equations can be written in the spectral representation. Then one applies the Schwartz inequality to derive the relation

$$\tau_g \geqslant \nu_g^{-1}. \tag{3.8}$$

Let $q(\boldsymbol{k})$ be the Fourier transform of the q_i variables, namely

$$q(\boldsymbol{k}) = N^{-1/2} \sum_{i=1}^{N} \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i) q_i. \tag{3.9}$$

If we put $g = q(\boldsymbol{k})$, τ_g is the *relaxation time of the system* for the appropriate $\boldsymbol{k}$ ($\boldsymbol{k} = 0$ in our case). In order to use inequality (3.8), we must evaluate the initial rate ($\nu_{q(k)}$). The equal time correlation function is proportional to the static susceptibility χ_k, i.e.

$$C_{q(k)}(0) = k_B T \chi_k. \tag{3.10}$$

We have also that

$$\frac{\mathrm{d}}{\mathrm{d}t} C_{q(k)}(t)\bigg|_{t=0} = -N^{-1} \sum_{j,m} \exp[\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}_j - \boldsymbol{r}_m)] \langle q_j D_q q_m \rangle_{\mathrm{eq}}. \tag{3.11}$$

It is easy to show that $D_q q_m = \sum_{q'_m=1}^{Q} w_m(\{q\}, \{q\}'_m)[q_m - q'_m]$ from the definition (3.3) of D_q. Therefore (3.11) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} C_{q(k)}(t)\bigg|_{t=0} = -N^{-1} \sum_j \sum_{q'_j} \langle q_j w(\{q\}, \{q\}'_j)[q_j - q'_j] \rangle_{\mathrm{eq}} \tag{3.12}$$

and the inequality (3.8) as

$$\tau_{q(k)} \geqslant \frac{N k_B T \chi_k}{\sum_j \sum_{q'_j} \langle q_j [q_j - q'_j] w_j(\{q\}, \{q\}'_j) \rangle_{\mathrm{eq}}}. \tag{3.13}$$

Let us consider now the Kawasaki dynamics. The function ϕ defined in (3.1) is inserted into the master equation (2.6). Then we can write an equation similar to (3.2). But we must define a new operator $D_q^{(K)}$ as

$$D_q^{(K)} \phi(\{q\}, t) = \sum_{i=1}^{N} W_i(\{q\}, \{q\}'_i)[\phi(\{q\}, t) - \phi(\{q\}'_i, t)]. \tag{3.14}$$

Again properties (3.4) hold and the eigenvalues of $D_q^{(K)}$ are real and non-negative. Moreover one finds

$$\frac{\mathrm{d}}{\mathrm{d}t} C_{q(k)}(t)\bigg|_{t=0} = -2N^{-1}[1 - \cos(\boldsymbol{k}\cdot\boldsymbol{a})] \sum_m \langle q_m (q_m - q_{m+1}) W_m(\{q\}, \{q\}'_m) \rangle_{\mathrm{eq}}. \tag{3.15}$$

Here $\boldsymbol{a}$ is a vector between neighbouring sites. Finally, the inequality for the relaxation time of the system is given by

$$\tau_{q(k)} \geqslant \frac{N k_B T \chi_k}{2[1 - \cos(\boldsymbol{k}\cdot\boldsymbol{a})] \sum_m \langle q_m (q_m - q_{m+1}) W_m(\{q\}, \{q\}'_m) \rangle_{\mathrm{eq}}}. \tag{3.16}$$

Inequalities (3.13) and (3.16) are the essential results of this section. They will be used in the next two sections to derive the lower bounds for the z exponent.

4. The Glauber dynamics

4.1. The exponential transition rate

First let us consider the Glauber dynamics of the Potts model with the following transition rate:

$$w_i(\{q\},\{q\}_i') = \Gamma\left[\frac{P_{eq}(\{q\}_i')}{P_{eq}(\{q\})}\right]^{1/2}. \tag{4.1}$$

This rate satisfies the detailed balance condition (2.5) and has been used in the literature (Forgacs *et al* 1980). In order to find a lower bound for the critical dynamic exponent we must evaluate the expression

$$A = \sum_{i=1}^{N}\sum_{q_i'=1}^{Q} \langle (q_i^2 - q_i q_i')\, w_i(\{q\},\{q\}_i')\rangle_{eq}. \tag{4.2}$$

Using the transfer matrices L_i and M_i defined as

$$\begin{aligned} M_i(q_i, q_{i+1}) &= \exp(K_i \delta_{q_i, q_{i+1}}) \\ L_i(q_i, q_{i+1}) &= \exp\left(\frac{K_i}{2}\,\delta_{q_i, q_{i+1}}\right) \end{aligned} \tag{4.3}$$

we can express (4.2) as follows:

$$A = \frac{\Gamma}{Z}\sum_{j=1}^{N}\sum_{q_j, q_j'=1}^{Q}\sum_{q_{j-1}, q_{j+1}=1}^{N} (q_j^2 - q_j q_j')\, L_{j-1}(q_{j-1}, q_j')\, L_{j-1}(q_{j-1}, q_j)\, L_j(q_j', q_{j+1})$$

$$\times L_j(q_j, q_{j+1}) \prod_{i=j+1}^{j-2} M_i(q_i, q_{i+1}). \tag{4.4}$$

Here Z is the partition function of the model. It is easily seen that all transfer matrices commute. Therefore, they can be diagonalised simultaneously. The eigenvalues of the matrices L_i and M_j are given respectively by

$$\begin{aligned} &\lambda_{i,1} = \exp\left(\frac{K_i}{2}\right) + Q - 1 \\ &\lambda_{i,2} = \exp\left(\frac{K_i}{2}\right) - 1 \qquad\qquad (Q-1)\text{-fold degenerate} \\ &\Lambda_{j,1} = \exp(K_j) + Q - 1 \\ &\Lambda_{j,2} = \exp(K_j) - 1 \qquad\qquad (Q-1)\text{-fold degenerate.} \end{aligned} \tag{4.5}$$

The next step is to express the elements of the L_i and M_j matrices in terms of their respective eigenvalues and eigenvectors. For example, we have that

$$\prod_{i=j+1}^{i-2} M_i(q_i, q_{i+1}) = \sum_{n=1}^{Q} \prod_{i=j+1}^{i-2} \Lambda_{i,n}\, \phi_n(q_{i+1})\, \phi_n(q_{i-1}) \tag{4.6}$$

where $\phi_n(q_i)$ is the q_i element of the eigenvector ϕ_n. Obviously, the partition function can also be written in terms of the eigenvalues $\Lambda_{i,n}$. So, in the thermodynamic limit,

we obtain that

$$A=\sum_{j=1}^{N}\Gamma[\Lambda_{j,1}\Lambda_{j-1,1}]^{-1}\sum_{i,k,l,p=1}^{Q}\sum_{q_j,q_j'=1}^{Q}\sum_{q_{j-1},q_{j+1}=1}(q_j^2-q_jq_j')\lambda_{j-1,i}\lambda_{j,k}\lambda_{j,l}\lambda_{j-1,p}\phi_i(q_{j-1})\phi_p(q_{j-1})$$
$$\times\phi_1(q_{j-1})\phi_i(q_j')\phi_k(q_j')\phi_l(q_j)\phi_p(q_j)\phi_k(q_{j+1})\phi_1(q_{j+1})\phi_l(q_{j-1}). \tag{4.7}$$

Using the following relations

$$\phi_1(q)=Q^{-1/2}\qquad\text{for all } q$$
$$\sum_{q=1}^{Q}\phi_i(q)\phi_k(q)=\delta_{i,k} \tag{4.8}$$
$$\sum_{i=1}^{Q}\phi_i(q)\phi_i(q')=\delta_{q,q'}$$

we can write (4.7) as

$$A=\frac{\Gamma(Q^2-1)}{12Q}\sum_{j=1}^{N}\frac{(\lambda_{j-1,1}^2-\lambda_{j-1,2}^2)(\lambda_{j,1}^2-\lambda_{j,2}^2)}{\Lambda_{j-1,1}\Lambda_{j,1}} \tag{4.9}$$

where the eigenvalues are given by (4.5). Near zero temperature the behaviour of A is given by

$$A\sim\frac{\Gamma}{6QN}\sum_{j=1}^{N}\exp\left(\frac{-(K_{j-1}+K_j)}{2}\right). \tag{4.10}$$

Clearly this sum is dominated by $\exp[-(1/2)(K_g+K_{g-1})]$, the largest term appearing in the sum (4.10). Consequently (3.13) can be written as

$$\tau_{q(k)}\geqslant\frac{k_B T}{\Gamma}\xi^{1+(J_{g-1}+J_g)/2J_s}. \tag{4.11}$$

Assuming the scaling form (1.1) for the relaxation time, the following lower bound for the z exponent is obtained:

$$z\geqslant 1+\frac{J_{g-1}+J_g}{2J_s}. \tag{4.12}$$

When the smallest interaction (J_s) in the model has, at least, another J_s interaction as neighbouring bond, the expression above gives us a trivial lower bound ($z\geqslant 2$). However, for all models in which the smallest interactions have neighbouring bonds different from J_s, we have from (4.12) a non-universal critical dynamic exponent.

4.2. The large transition rate

Another interesting transition rate, which gives a Q-dependent dynamical exponent is the one used by Lage (1985), namely

$$w_i(\{q\},\{q\}_i')=\Gamma\frac{\exp(-K_{i-1}\delta_{q_{i-1},q_i'}-K_i\delta_{q_i',q_{i+1}})}{\sum_{q=1}^{Q}\exp(-K_{i-1}\delta_{q,q_{i-1}}-K_i\delta_{q,q_{i+1}})}. \tag{4.13}$$

Again, we must evaluate (4.2) by the transfer matrix technique. The matrix M has the same definition as before but now we must define a new matrix L which is given

by $L_i(q_i, q_{i+1}) = \exp(-K_i\delta_{q_i,q_{i+1}})$. The eigenvalues of L are $\lambda_{i,1} = \exp(-K_i) + Q - 1$ and $\lambda_{i,2} = \exp(-K_i) - 1$ (Q-fold degenerate). In the same lines as before we find that

$$A = \frac{\Gamma Q^2(Q-1)}{12} \sum_{j=1}^{N} \left(\frac{[\exp(-K_{j-1} - K_j) + Q - 1]^{-1}}{[\exp(K_{j-1}) + Q - 1][\exp(K_j + Q - 1]} + \frac{(Q-1)[\exp(-K_{j-1}) + \exp(-K_j) + Q - 2]^{-1}}{[\exp(K_{j-1}) + Q - 1][\exp(K_j + Q - 1]} \right). \tag{4.14}$$

In this formula we can see that the behaviour near $T = 0$ of the case $Q = 2$ is different from the case with $Q > 2$, because of the last term of (4.14). When $Q = 2$ the behaviour of A near the critical temperature is given by

$$A \sim \frac{\Gamma}{N} \sum_{j=1}^{N} [\exp(K_{j-1}) + \exp(K_j)]^{-1}. \tag{4.15}$$

We can see that A is dominated by the smallest term $\exp(K_g) + \exp(K_{g-1})$. If $J_g > J_{g-1}$, we can write (3.13) as

$$\tau_{q(0)} \geqslant \xi^{1+(J_g/J_s)}. \tag{4.16}$$

Then one obtains that $z \geqslant 1 + (J_g/J_s)$. In the models where the neighbouring interactions of the smallest ones (J_s) are different from J_{s1} we obtain a non-universal behaviour. It is worth mentioning that for the isotropic case ($J_i = J$ for all i) and for the alternating bond model ($J_{2i} = J_s$ and $J_{2i+1} = J_1$), the lower bounds coincide with the exact results ($z = 2$ and $z = 1 + (J_1/J_s)$, respectively (Droz *et al* 1986a)).

For $Q > 2$, the equation (4.14) can be written near $T = 0$ as

$$A \sim \sum_{j=1}^{N} \exp(-K_j - K_{j-1}). \tag{4.17}$$

Therefore we can obtain the following lower bound for the critical dynamic exponent:

$$z \geqslant 1 + \frac{J_g + J_{g-1}}{J_s} \tag{4.18}$$

where $J_{g-1} + J_g$ is the smallest sum of all pairs of next-neighbouring interactions in the chain. So we prove the non-universal behaviour for all models which do not have the sequence J_sJ_s in the basic cell.

5. The Kawasaki dynamics

5.1. The exponential transition rate

Let us now concentrate our attention on the Kawasaki dynamics with the transition rates defined by

$$W_i(\{q\}, \{q\}_i') = \Gamma(1 - \delta_{q_i,q_{i+1}}) \exp\left(\frac{K_{i-1}}{2} (\delta_{q_{i-1},q_{i+1}} - \delta_{q_{i-1},q_i}) + \frac{K_{i+1}}{2} (\delta_{q_i,q_{i+2}} - \delta_{q_{i+1},q_{i+2}}) \right). \tag{5.1}$$

This rate has also been used in the literature (Droz *et al* 1986b). We must evaluate the expression

$$B = \sum_{j=1}^{N} \langle W_j(\{q\}, \{q\}_j')[q_i^2 - q_iq_{i+1}] \rangle_{eq} \tag{5.2}$$

in order to obtain a lower bound for the z exponent by using inequality (3.16). Again we must use the transfer matrix technique. Following the same lines as in the previous section, we obtain

$$B=\frac{\Gamma(Q^2-1)}{6NQ}\sum_{j=1}^{N}\frac{(\lambda_{j-1,1}^2-\lambda_{j-1,2}^2)(\lambda_{j+1,1}^2-\lambda_{j+1,2}^2)(\Lambda_{j,1}-\Lambda_{j,2})}{\Lambda_{j-1,1}\Lambda_{j,1}\Lambda_{j+1,1}}. \tag{5.3}$$

Here the eigenvalues are given by (4.5). Near zero temperature is easy to find the behaviour of B, namely

$$B\approx N^{-1}\sum_{j=1}^{N}\left[\exp\left(K_j+\frac{K_{j+1}}{2}+\frac{K_{j-1}}{2}\right)\right]^{-1}. \tag{5.4}$$

If J_g, $(J_{g-1}/2)$, $(J_{g+1}/2)$ are the smallest three neighbouring bonds in the model, B has a behaviour near the critical temperature dominated by $\exp[-K_g-(K_{g-1}/2)-(K_{g+1}/2)]$. Therefore assuming the scaling behaviour (1.1) for the relaxation time, we find from inequality (3.16) that

$$z\geqslant 3+\frac{2J_g+J_{g-1}+J_{g+1}}{2J_s}. \tag{5.5}$$

So, for all models that do not have three adjacent smallest interactions, the z exponent is non-universal.

5.2. The Kawasaki transition rate

Let us consider now the transition rates introduced by Kawasaki, namely

$$W_i(\{q\},\{q\}_i')=2\Gamma(1-\delta_{q_i,q_{i+1}})[1+\exp\{-K_{i-1}(\delta_{q_{i-1},q_{i+1}}-\delta_{q_i,q_{i-1}})-K_{i+1}(\delta_{q_i,q_{i+2}}-\delta_{q_{i+1},q_{i+2}})\}]^{-1}. \tag{5.6}$$

Following the same lines as before, we find after some algebra that near $T=0$, B behaves as

$$B\approx N^{-1}\sum_{j=1}^{N}\exp(-K_j-K_u) \tag{5.7}$$

where $K_u=K_{j-1}$ if $K_{j-1}>K_{j+1}$ or $K_u=K_{j+1}$ in the other case. Then we can obtain that

$$z\geqslant 3+\frac{J_g+J_u}{J_s} \tag{5.8}$$

where $\exp(J_g+J_u)$ is the smallest term in the sum (5.7). The non-universal behaviour is proved for all models without the sequence J_s, J_s, J_s in the chain.

5.3. The large transition rate

The last step is to study the extension to Kawasaki dynamics of the Lage transition rate (Droz *et al* 1986b). It can be defined as

$$W_i(\{q\},\{q\}_i')=\frac{\Gamma(1-\delta_{q_i,q_{i+1}})\exp(-K_{i-1}\delta_{q_{i-1},q_i}-K_{i+1}\delta_{q_{i+1},q_{i+2}})}{\Sigma_q\,\Sigma_{q'}\exp(-K_{i-1}\delta_{q_{i-1},q}-K_{i+1}\delta_{q_{i+2},q'})}. \tag{5.9}$$

Following the same steps as before, we find after some algebra that

$$z \geqslant 3 + \frac{J_g + J_{g-1} + J_{g+1}}{J_s}. \tag{5.10}$$

In this case the non-universal behaviour is proved for all models that do not have three adjacent J_s bonds in the basic cell. Note that for the isotropic case ($J_i = J$ for all i) we obtain that $z \geqslant 6$. This result does not agree with the one found by Droz *et al* (1986b) namely $z = 5$, by domain wall arguments. This point will be made clearer in the next section.

6. The domain wall argument for the alternating bond models

The results obtained in the last two sections can be reproduced for the alternating bond models by a simple physical argument about the movement of domain walls. The argument has been proposed for the Ising model (Cordery *et al* 1981) and has been also used in the study of the homogeneous Potts model (Droz *et al* 1986b). The behaviour of the relaxation time near $T = 0$ is determined by the time (τ_w) it takes for a domain wall to move a distance ξ in the *fastest way*. Computing this time by random walk arguments we can find the z exponent using $\tau_w \sim \xi^z$. If the chosen mechanism of motion of the wall is the fastest possible then the resulting value of z should be an upper bound to the exact one (Cordery *et al* 1981). As a matter of fact, this approach leads to the exact values of z for all one-dimensional cases for which the answer is known.

Let us consider first the Glauber dynamics of the alternating bond model (J_1, J_s, J_1, J_s ...) with the transition rates defined by (4.1). One can see by inspection that the domain wall movements are still the fastest mechanism. A domain wall makes two steps in a time given on average by $\{1/w^{(1)}\} + \{1/w^{(2)}\}$. Here $w^{(1)}$ ($w^{(2)}$) corresponds to the flipping rate of the spins at the domain wall when we have a J_1 (J_s) interaction between the two domains. To move a distance ξ, the wall must make $\xi^2/2$ steps. Then, it is easy to find that

$$\tau_w = \frac{\xi^2}{2}\left(\frac{1}{w^{(1)}} + \frac{1}{w^{(2)}}\right). \tag{6.1}$$

From (4.1) we find that near the zero temperature, the behaviour of those rates is given by

$$\begin{aligned} w^{(1)} &\approx \Gamma \exp[-\tfrac{1}{2}(K_1 - K_s)] \\ w^{(2)} &\approx \Gamma \exp[-\tfrac{1}{2}(K_s - K_1)]. \end{aligned} \tag{6.2}$$

Therefore the z exponent, given by $z = [3 + (J_1/J_s)]/2$, is equal to the lower bound of (4.12).

Let us now consider the Lage transition rates (4.13). Again, the fastest mechanisms are the wall movements and $w^{(1),(2)}$ are given by

$$\begin{aligned} w^{(1)} &\approx \Gamma \frac{\exp(-K_s)}{\exp(-K_s) + \exp(-K_1) + Q - 2} \\ w^{(2)} &\approx \Gamma \frac{\exp(-K_1)}{\exp(-K_s) + \exp(-K_1) + Q - 2}. \end{aligned} \tag{6.3}$$

If $Q=2$, we find that $z=1+(J_1/J_s)$. This value coincides with the lower bound (4.16) and with the exact value (Droz *et al* 1986a). In the other cases ($Q>2$), we obtain that $z=2+(J_1/J_s)$. This value is equal to the lower bound (4.18).

The domain wall argument can also be applied to the Kawasaki dynamics. In this case, the domain walls do not move independently. So we must consider the movement of the spins. Near zero temperature and at long times two situations are important in dynamics. In the first case the interaction between two domains is J_1 (see figure 1(a)); in the other one the interaction is J_s. Let us discuss in details the first situation, which is depicted in figure 1. The discussion of the second situation is similar. Firstly, the spins at the domain wall (spins i and $i+1$ in figure 1(a)) are exchanged with a rate $W^{(1)}=W_i(\{q\},\{q\}_i')$. Then we have the situation depicted in figure 1(b), in which the exchange rate W_i between spins i and $i+1$ is greater than W_{i+1}, the exchange rate between spins $i+1$ and $i+2$. So the probability y_1 that the up spin gives the first step directed to the middle of the down spin domain is given by $y_1=W_{i+1}/(W_i+W_{i+1})$. Obviously, the probability that the first step be in direction of the nearest domain wall is $1-y_1$. After the exchange between spins $i+1$ and $i+2$ in figure 1(b), we have an up spin performing a random walk in a domain of down spins (see figure 1(c)). Thus the up spin moves through the domain and comes out at the other side. For a domain of size ξ this happens with probability $P_\xi^{(1)}\sim[y_1/(1-y_1)]\xi^{-1}$ (Harris 1983). If the first spin of the domain is exchanged at the wall with a rate $W^{(1)}$, the second spin will be exchanged with a rate $W^{(2)}$. This corresponds at the second situation, in which we

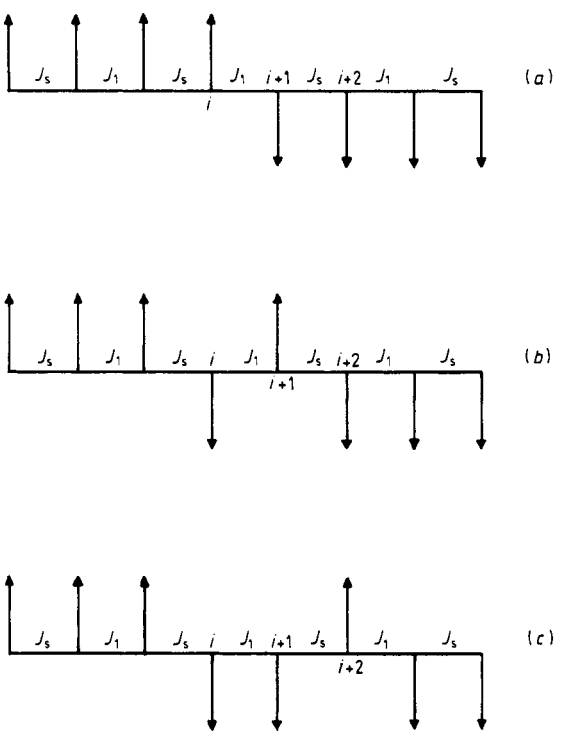

Figure 1. Important configurations for the domain wall argument applied to Kawasaki dynamics. (a) The spins i and $i+1$ are exchanged with a small rate $W^{(1)}$; (b) the exchange rate between spins i and $i+1$ is greater than the one between spins $i+1$ and $i+2$; (c) the up spin performs a random walk.

have a J_s interaction between two domains. Thus the whole domain has moved two steps in an average time $[1/(P_\xi^{(1)}W^{(1)})+1/(P_\xi^{(2)}W^{(2)})]$, where $P_\xi^{(2)}\sim[y_2/(1-y_2)]\xi^{-1}$. To move a distance ξ, the domain wall must make ξ^2 steps. Therefore we have that

$$\tau_w \approx \tfrac{1}{2}\xi^2\left[\frac{1}{P_\xi^{(1)}W^{(1)}}+\frac{1}{P_\xi^{(2)}W^{(2)}}\right]. \tag{6.4}$$

Considering the transition rates given by (5.1) we have that $y_2=\xi^{-(J_1/J_s)}$ and $y_1=\xi^{-1}$. Evaluating $W^{(1)}$ and $W^{(2)}$ it is easy to find that

$$z=3+2J_1/J_s. \tag{6.5}$$

The lower bound (5.5), given by $z_{LB}=4+(J_1/J_s)$, is smaller than the exponent found with the domain wall argument for $J_1>J_s$. But the value (6.5) would be the exact value. It is worth mentioning that in Kamphorst Leal da Silva (1986) a wrong exponent ($z=3+(J_1/J_2)$) has been found for the Ising model because the probability y_a has not been considered.

In the case of the Lage transition rates (5.9), we find that

$$z=4+2J_1/J_s. \tag{6.6}$$

Here we have that $y_1\sim\xi^{-(J_1/J_s)}$ and $y_2\sim\xi^{-1}$. Since this probability has been considered as a constant independent of the temperature by Droz *et al* (1986b), a wrong exponent has been found for the isotropic model ($z=5$ instead of $z=6$). Note that in this problem an up spin in the middle of a down spin domain exchanges with a non-symmetrical rate proportional to ξ^{-1}. This means that the up spin takes a long time to traverse the domain. However, this time is not long enough to invalidate the domain wall argument.

Finally let us consider the Kawasaki rates (5.6). In this case the probability y_a is not important because it has a constant value ($y_a=\frac{1}{3}$). We find, after some algebra, that the z axis exponent is given by

$$z=3+2J_1/J_s. \tag{6.7}$$

This value is greater than the lower bound ($z_{LB}=4+(J_1/J_s)$) of expression (5.8) but it would agree with the exact value.

7. Conclusions

The relaxational and diffusive kinetic Potts model in one dimension has been studied for several transition rates. The main conclusion is that the dynamical exponent depends on the sptatial non-uniformity of the coupling strengths. From the initial response rate of the system and the scaling behaviour of the relaxation time, rigorous lower bounds for the z exponent have been established. We have obtained essentially that

$$\tau_k \approx \xi^z \geqslant \frac{k_B T\chi_k}{A} \tag{7.1}$$

where A is the kinetic coefficient given by (4.2) for Glauber dynamics and by (5.2) for Kawasaki dynamics. Dynamic non-universality arises from the nonuniversal vanishing of this coefficient, which depends on local equilibrium correlations. A goes to zero only because the critical temperature is $T_c=0$. The non-universal exponent is therefore

a consequence of zero critical temperature. In dimensions greater than one we do not expect non-universal behaviour because the kinetic coefficients are probably finite. In this case (7.1) gives us only the inequalities $z \geq \gamma/\nu$ for the Glauber dynamics and $x \geq 2+(\gamma/\nu)$ for the Kawasaki one. It is worth mentioning that the initial response can be used efficiently in one-dimensional periodic models to prove non-universal dynamical behaviour. For disordered models, however, we obtain only a trivial lower bound. This happens because sequences of interactions, which are forbidden in our proof (for example, the J_s, J_s, J_s sequence), certainly appears when the disorder average is made.

For the alternating bond model, the non-universal behaviour has been explained by the domain wall argument, a simple microscopic mechanism. For Glauber dynamics the z exponent found by this argument is equal to the lower bound of the initial response. This does not happen for the Kawasaki dynamics when different interactions are present in the basic cell. It could be interesting to do Monte Carlo simulation in order to decide if the z exponent found by the domain wall argument is the exact one.

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